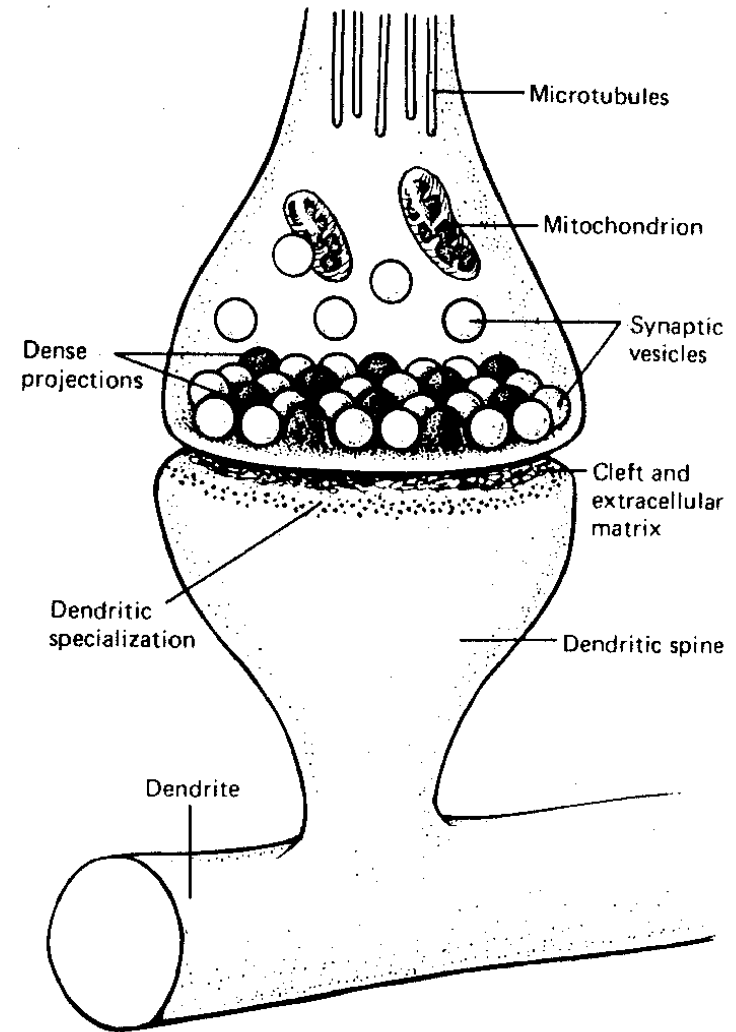
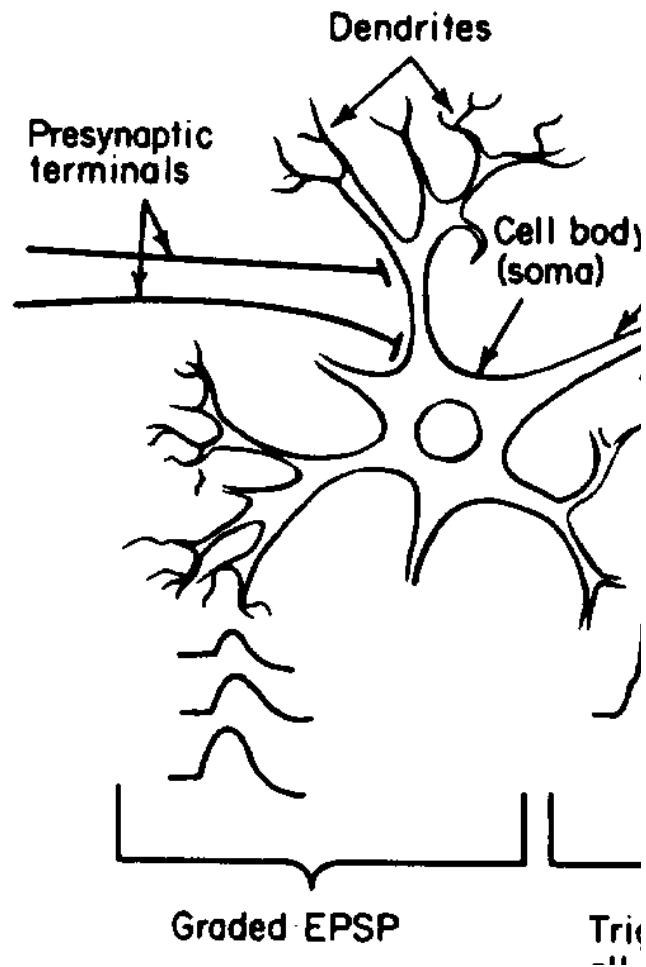
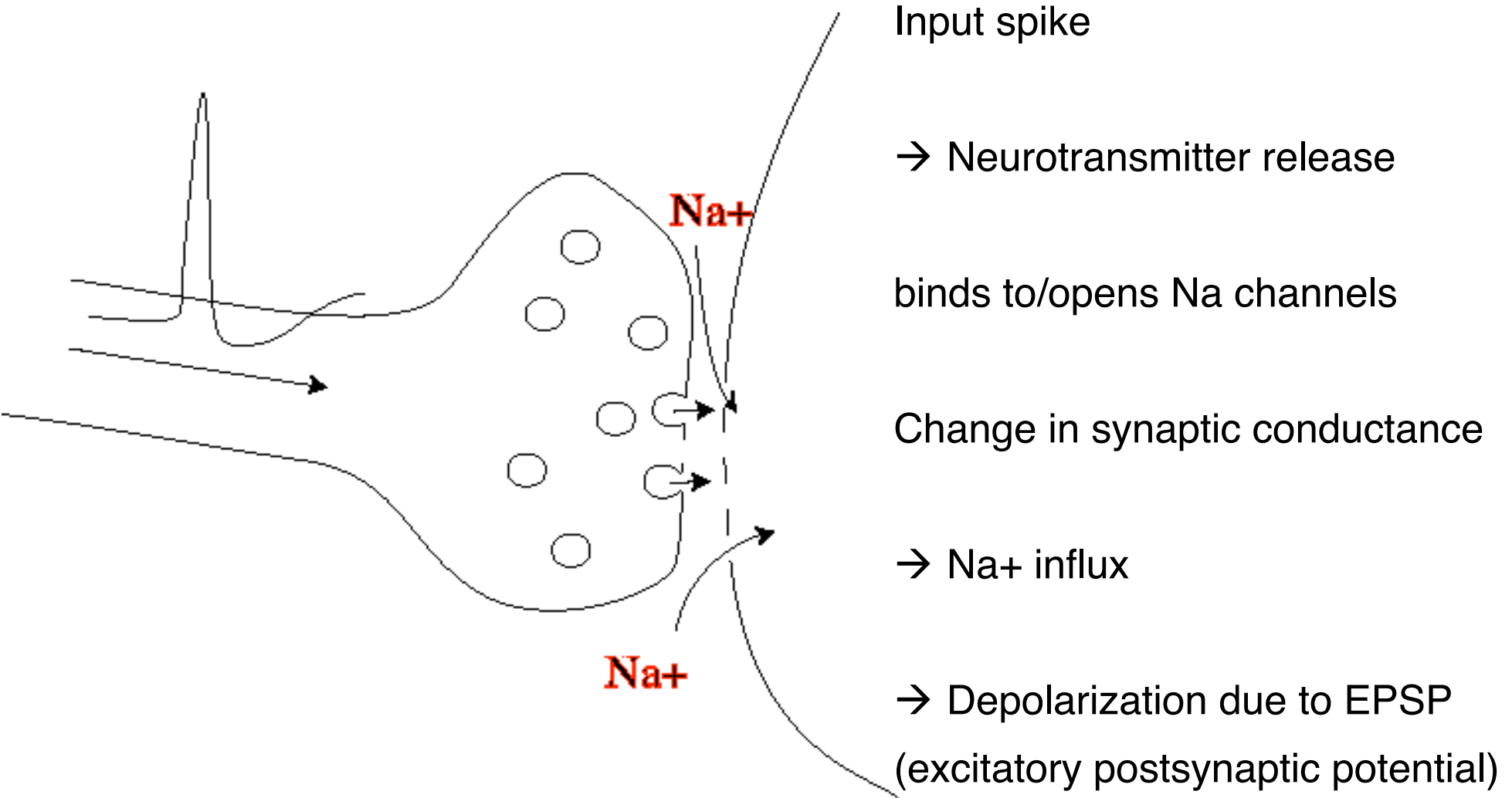


Neurons to networks

How do synapses transform inputs?



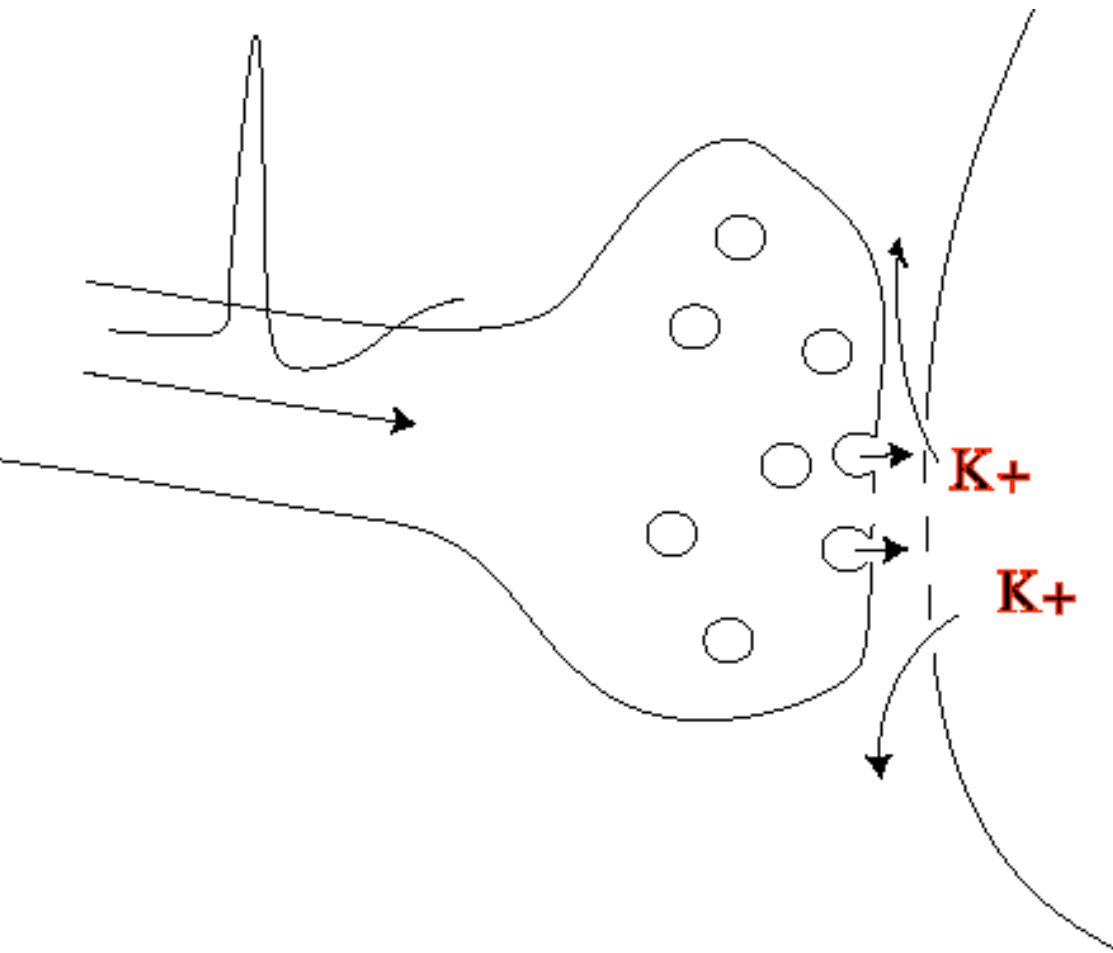
Excitatory synapse



E.g. AMPA synapse

Vocab: Depolarization means make V less neg = more positive

Inhibitory synapse



Input spike

→ Neurotransmitter release

binds to/opens K channels

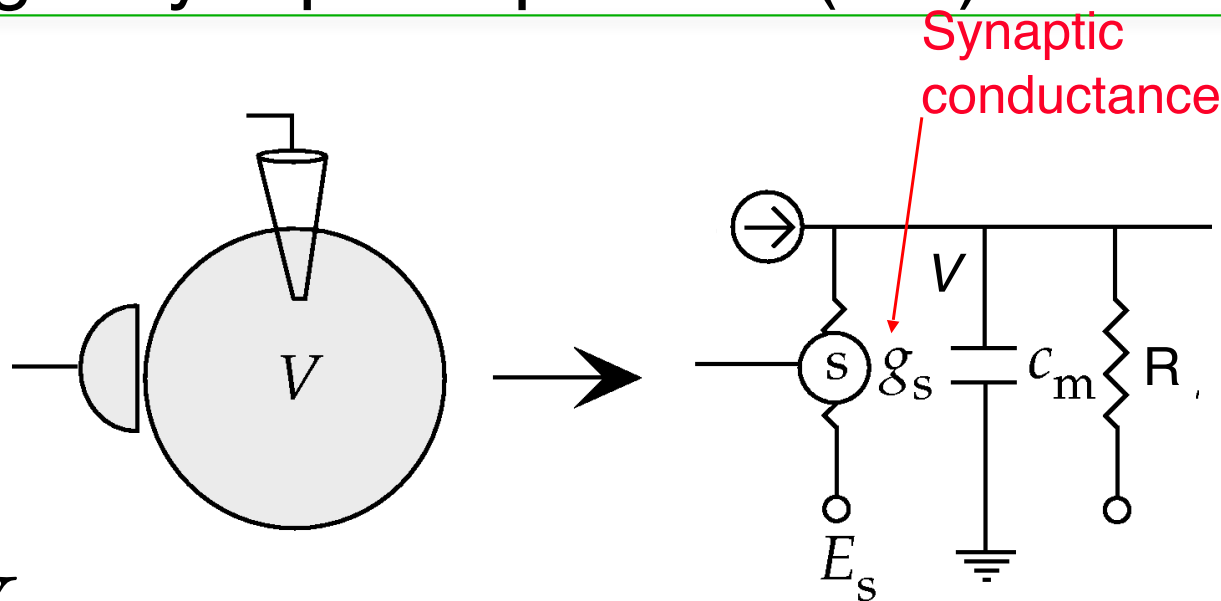
Change in synaptic conductance

K⁺ leaves cell

→ Hyperpolarization due to IPSP
(inhibitory postsynaptic potential)

Vocab: hyperpolarization means make V more negative

Modeling a synaptic input to a (RC) neuron



$$C \frac{dV}{dt} = g_L (E_L - V) + g_s(t) [E_s - V]$$

$g_s(t)$ synaptic conductance

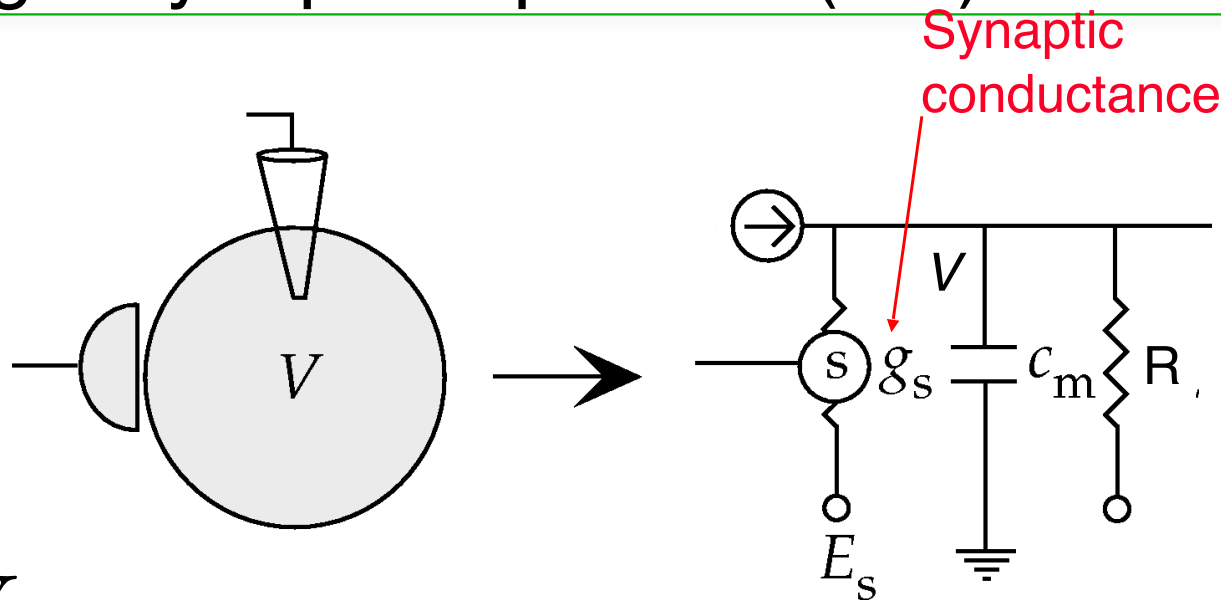
E_s synaptic reversal potential

$E_s > V_{threshold} \rightarrow$ Excitatory

$E_s < E_L \rightarrow$ Inhibitory

$E_s \approx E_L \rightarrow$ Shunting

Modeling a synaptic input to a (RC) neuron



$$C \frac{dV}{dt} = g_L (E_L - V) + g_s(t) [E_s - V]$$

$g_s(t)$ synaptic conductance

$$g_s = g_{s,max} P_{rel} P_s \leftarrow \begin{array}{l} \text{Probability of postsynaptic channel opening} \\ (= \text{fraction of channels opened}) \end{array}$$

\swarrow
Probability of transmitter release given an input spike

Basic synapse model

Assume $P_{\text{rel}} = 1$ (for now)

What does a single spike input do to P_s ?

Kinetic model:

closed $\xrightarrow{\alpha_s}$ open

open $\xrightarrow{\beta_s}$ closed

$$\frac{dP_s}{dt} = \alpha_s (1 - P_s) - \beta_s P_s$$

Opening rate α_s (indicated by a blue arrow from α_s to the text)

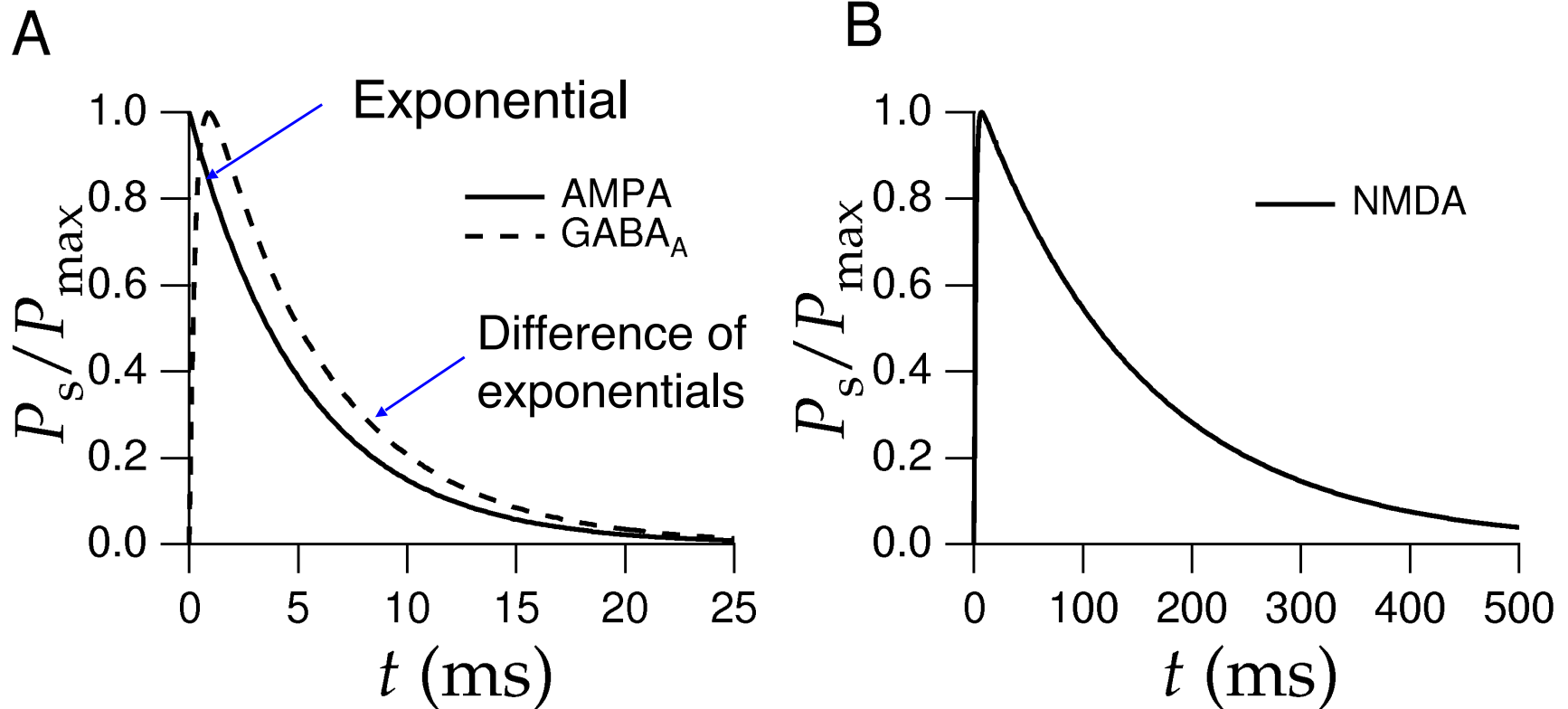
Fraction of channels closed $(1 - P_s)$ (indicated by a blue arrow from $(1 - P_s)$ to the text)

Closing rate β_s (indicated by a blue arrow from β_s to the text)

Fraction of channels open P_s (indicated by a blue arrow from P_s to the text)

Where: $\alpha_s(V(t), Ca(t), \dots)$ $\beta_s(V(t), Ca(t), \dots)$

Synaptic filters

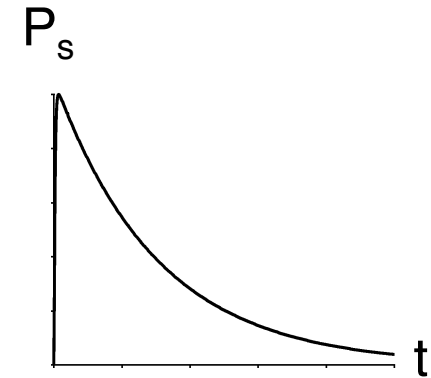


A difference of exponentials model better fits biological data for GABA, NMDA synapse types

Simplified synaptic models

Difference of exponentials:

$$P_s(t) = \text{const} \cdot P_{\max} \left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}} \right)$$

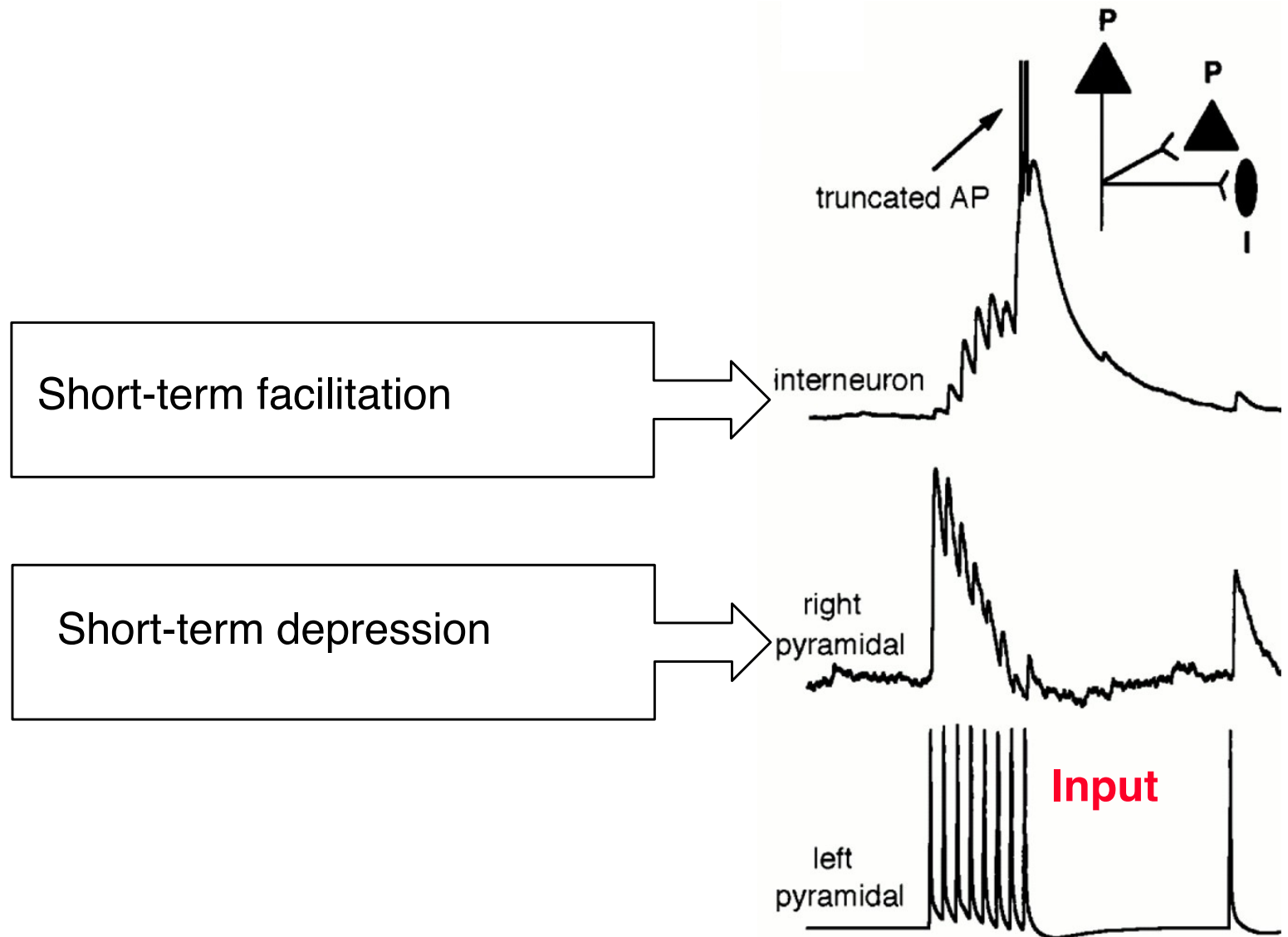


Alpha function:

$$P_s(t) = \text{const} \cdot \frac{t}{\tau_{peak}} e^{-\frac{t}{\tau_{peak}}}$$

What happens with a sequence of input spikes?

- Biological synapses are dynamic!



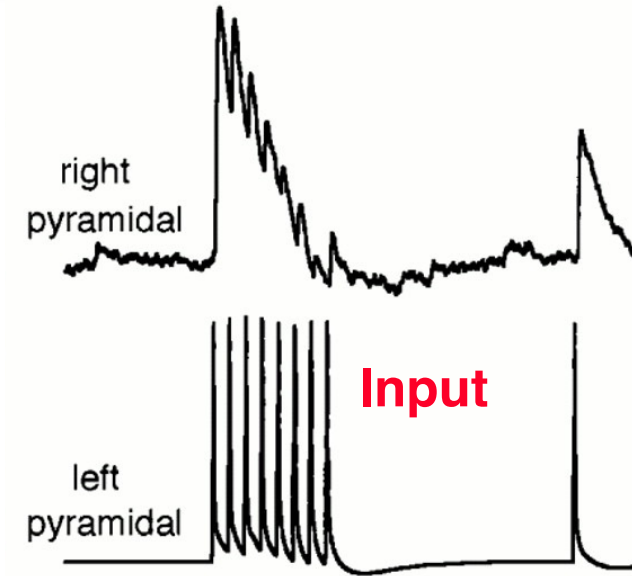
(Markram & Tsodyks, 1998)

Short-term synaptic plasticity: describe this via P_{rel}

Recall definition of synaptic conductance:

$$g_s = g_{s,max} P_{rel} P_s$$

Idea: Specify how P_{rel} changes as a function of consecutive input spikes



$$\tau_P \frac{dP_{rel}}{dt} = P_0 - P_{rel}$$

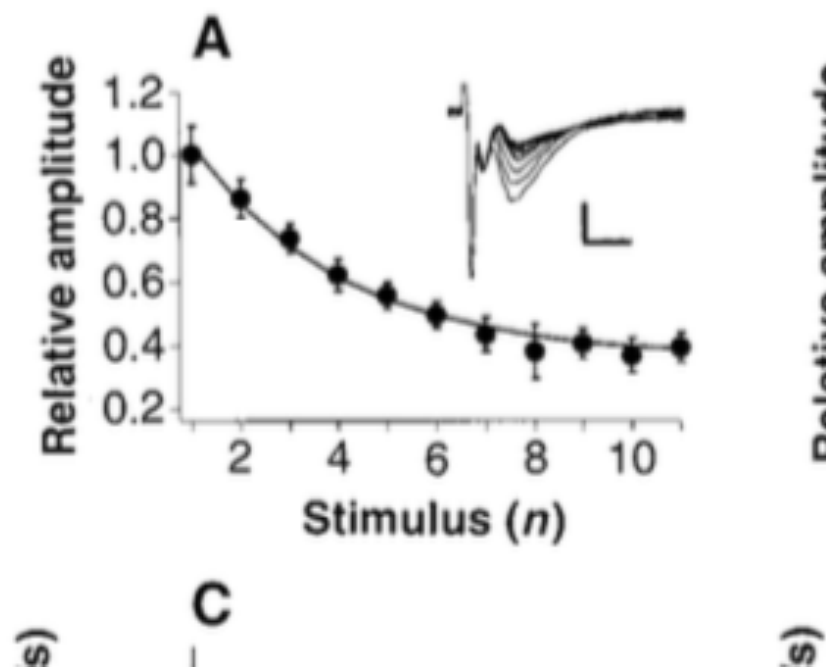
Between input spikes, P_{rel} decays exponentially back to P_0

If input spike:

$$P_{rel} \rightarrow f_D P_{rel}$$

depression: decrement P_{rel}

Fig. 1. Experimental results and fits of the model for synaptic depression. **(A)** Depression of synaptic responses during repetitive stimulation. Filled circles indicate normalized average field potential amplitudes evoked by 11 consecutive stimuli at 20 Hz. Error bars are stan-



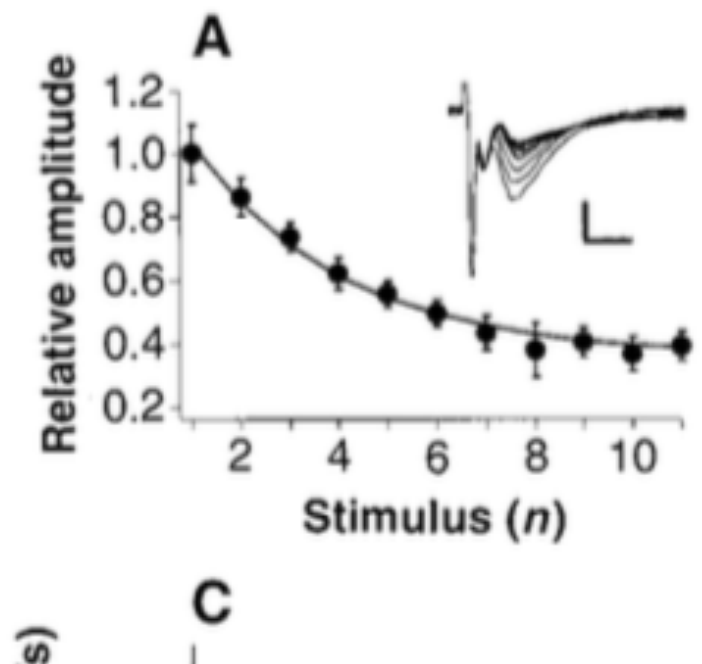
Lab exercise:

Write a code that implements the Abbott et al mechanism for synaptic depression.

Drive the synapse with spikes occurring regularly at 20 Hz, as in Fig. 1A of Abbott et al '97. Can you reproduce that figure?

Hint: this should be a few lines of code.

Fig. 1. Experimental results and fits of the model for synaptic depression. **(A)** Depression of synaptic responses during repetitive stimulation. Filled circles indicate normalized average field potential amplitudes evoked by 11 consecutive stimuli at 20 Hz. Error bars are stan-



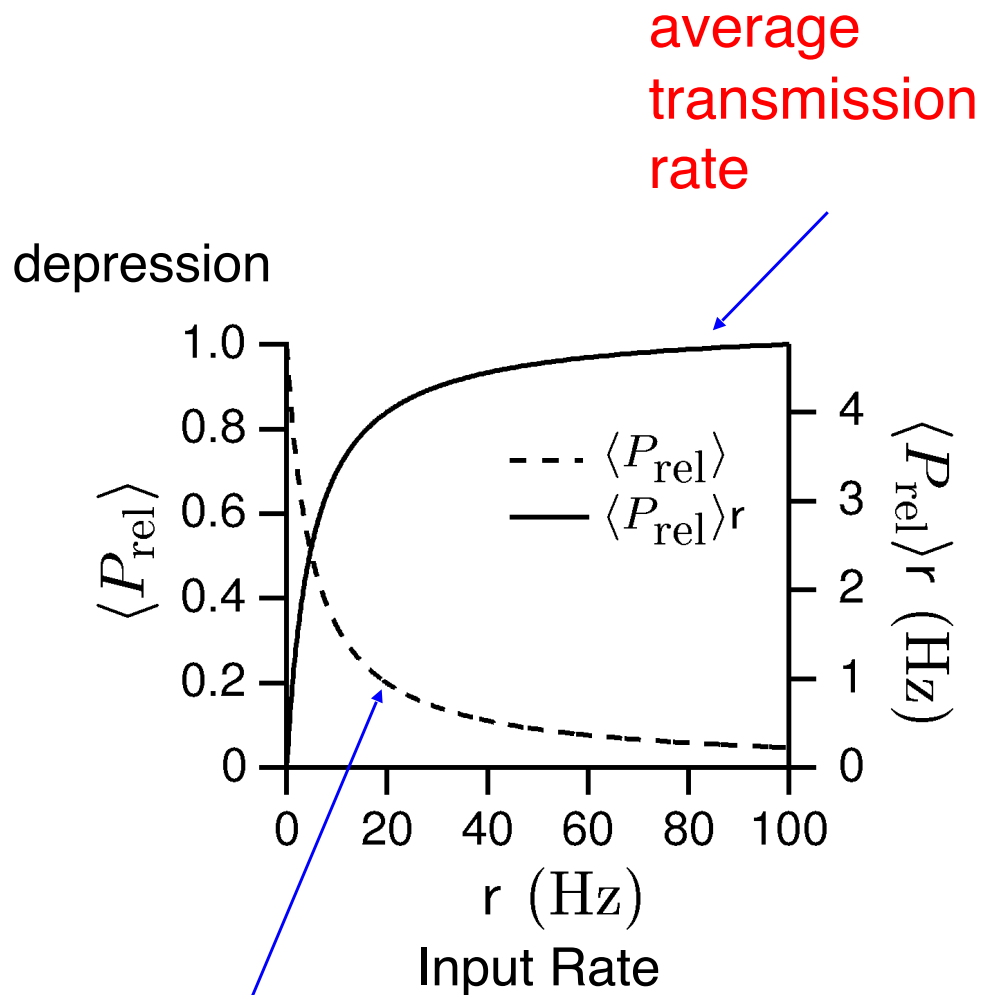
Impact of synaptic depression

Key result (a few lines of calculation, see (7) in paper [ESB notes]):

If synapse receives input spikes at rate r , then the **steady state value** of

$$P_{\text{rel}}(r) \sim 1/r$$

Consequences of synaptic depression: steady state



average
release
probability

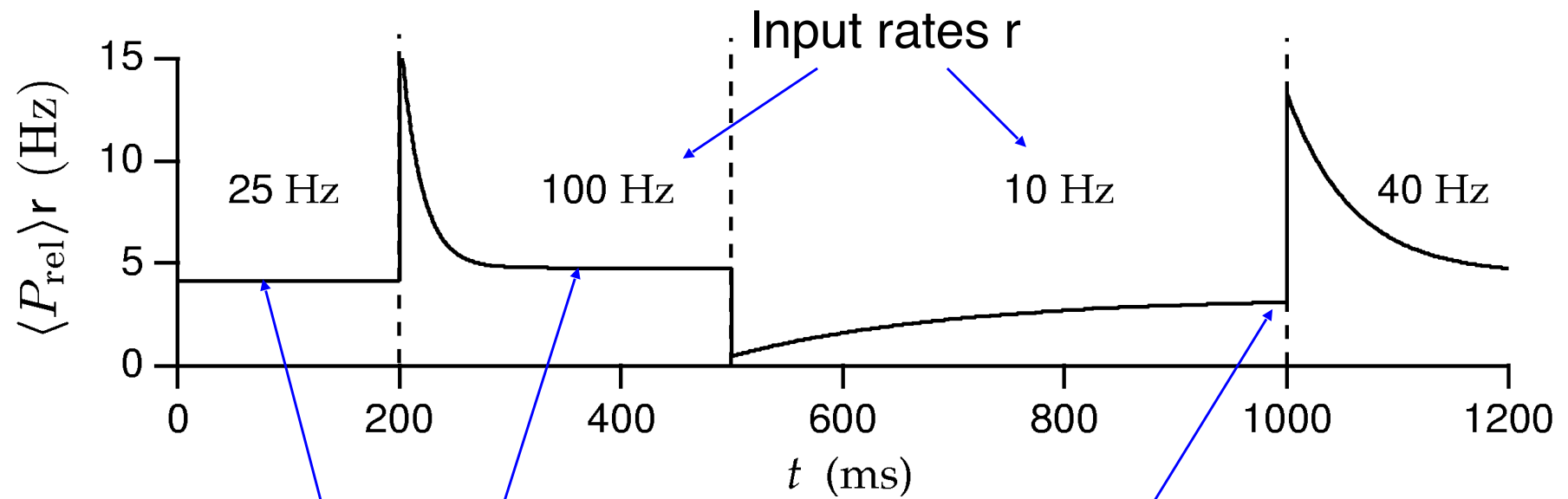
At steady state,

$$P_{rel}(r) \times r \approx \text{const for large } r$$

Constant synaptic input for
Wide range of inputs!

Steady-state gain control

Consequences of synaptic depression: dynamic response



Steady-state transmission rates are similar for different rates

Transient inputs are amplified relative to steady-state inputs

In fact: an equal-percent change from baseline gives an equal transient response.

Who cares? Abbott et al 97: Neuron gets inputs from 1000's of upstream cells, each of which fires at 1-200 Hz. How can we be responsive to all?

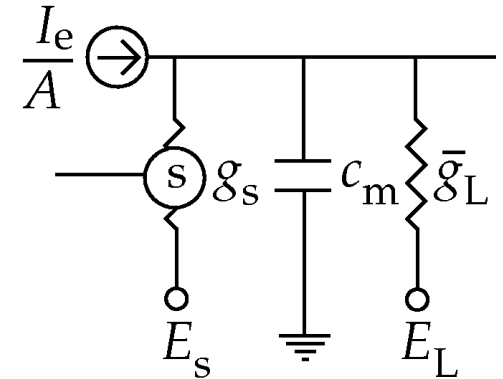
Synaptic depression yields "gain control" to satisfy this.

vs. Adaptation or inhibition, does so in an INPUT-SPECIFIC way!

Extending the model to include facilitation

Recall definition of synaptic conductance:

$$g_s = g_{s,\max} P_{rel} P_s$$



If input spike:

$$P_{rel} \rightarrow f_D P_{rel}$$

depression: decrement P_{rel}

$$P_{rel} \rightarrow P_{rel} + f_F (1 - P_{rel})$$

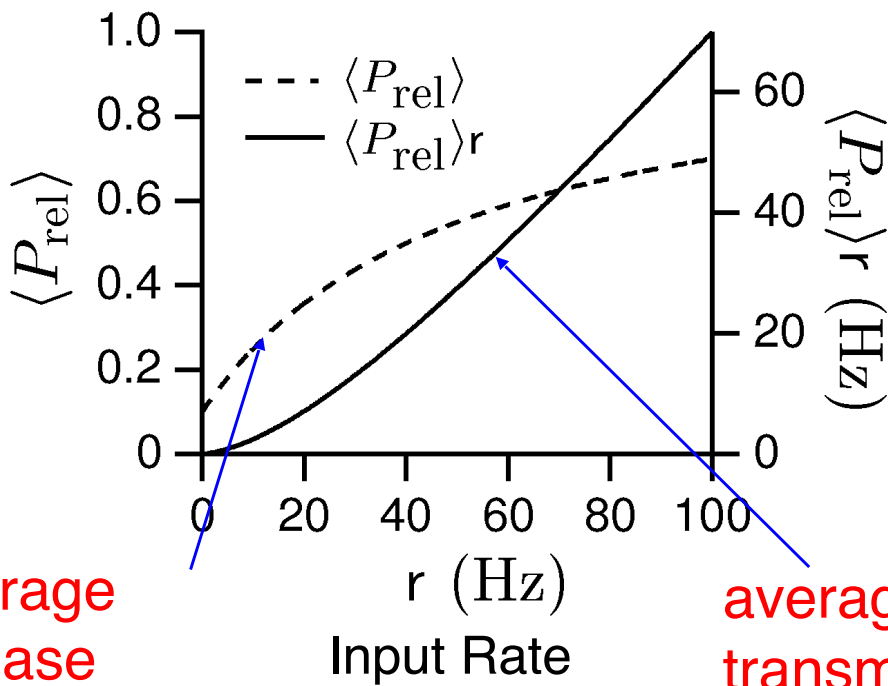
facilitation: increment P_{rel}

Between input spikes, P_{rel} still decays exponentially back to P_0

Abbott et al 1997

Effects of synaptic facilitation & depression

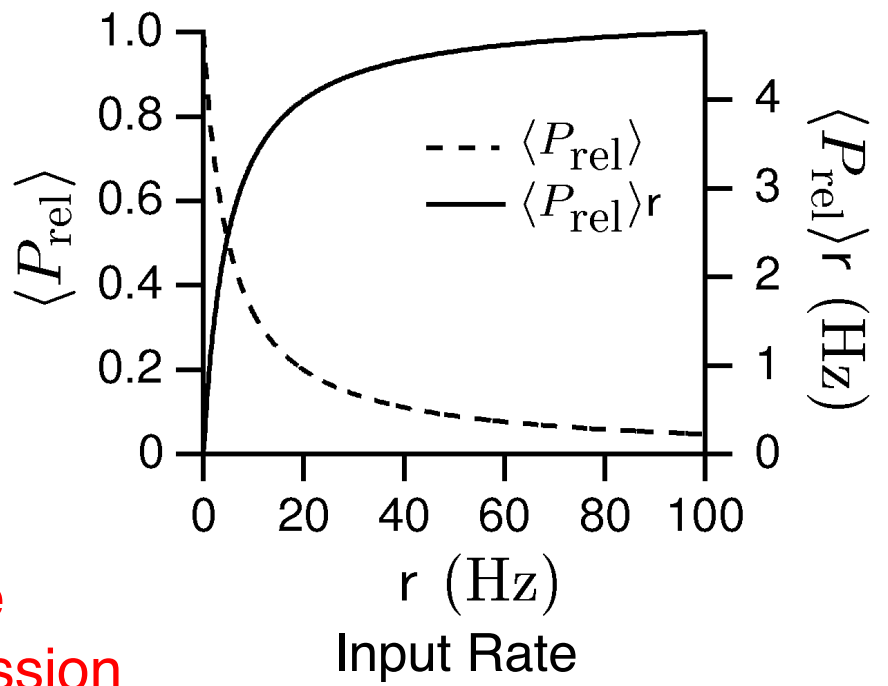
facilitation



average
release
probability

average
transmission
rate

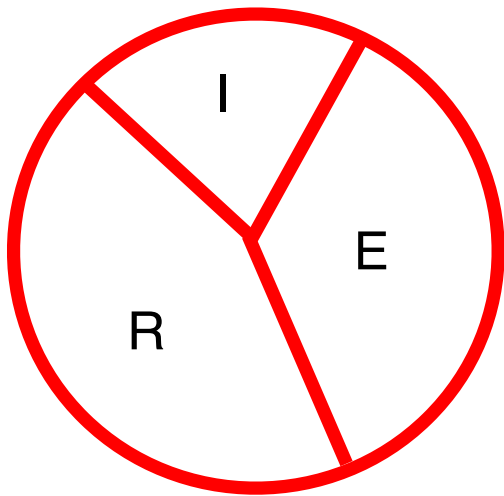
depression



$\langle P_{rel} \rangle r$ (Hz)

More detailed plasticity model

- Incoming spike activates a fraction of recovered resources R to become effective (E)
- Amount of effective resources E govern size of $g_s(t)$
- Effective resources rapidly inactivate (msec)
- Inactivated recover (100s of msec)
- $E(t)$ determines postsynaptic current



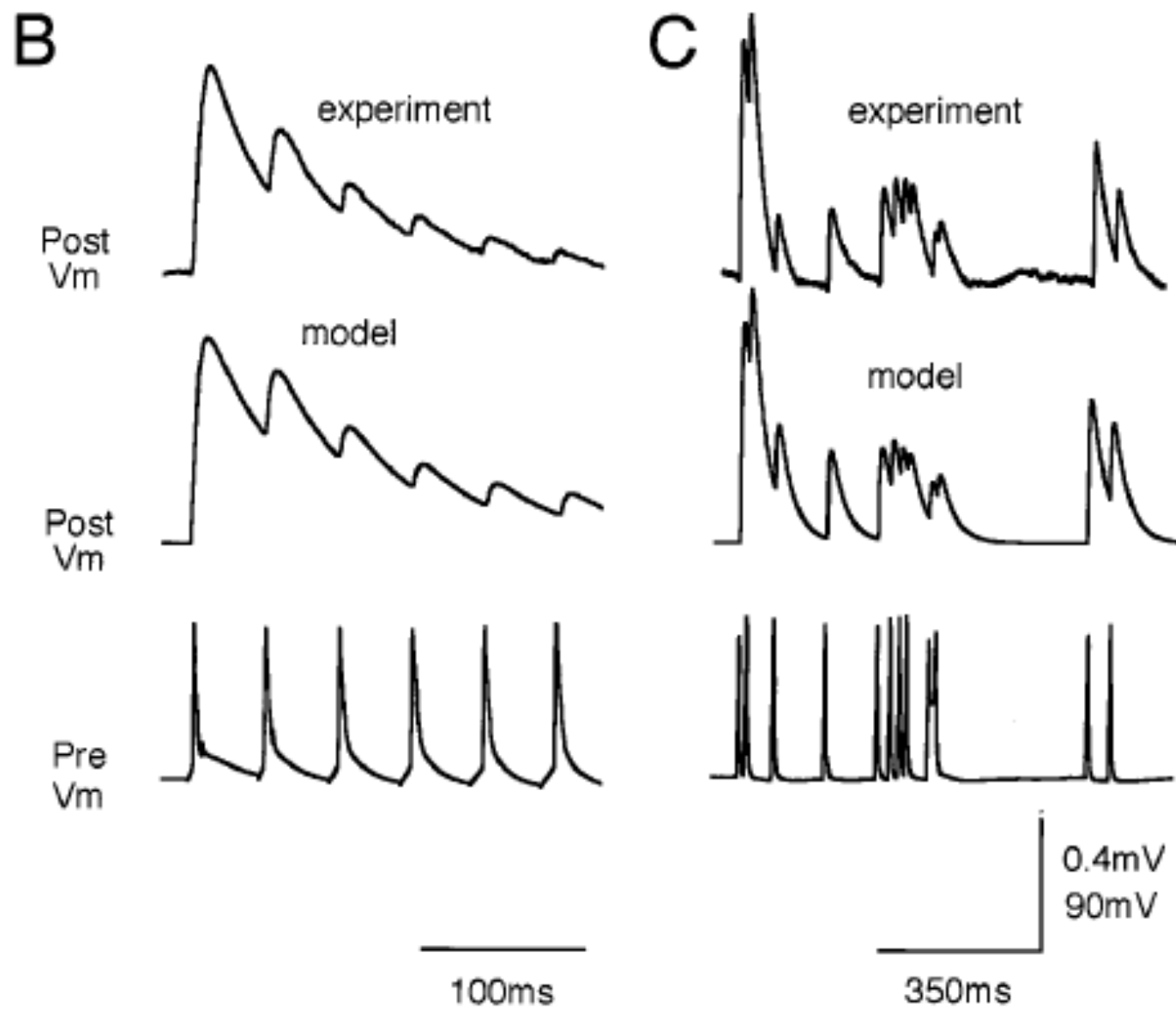
t_{AP} : time of spike
at all spike times
jump R by $-U_{SE} * R(t_{AP})$

$$\frac{dR}{dt} = \frac{I}{\tau_{rec}} - \underbrace{U_{SE} \cdot R \cdot \delta(t - t_{AP})}$$

$$\frac{dE}{dt} = -\frac{E}{\tau_{inact}} + \underbrace{U_{SE} \cdot R \cdot \delta(t - t_{AP})}_{\text{similar meaning}}$$

$$I = 1 - R - E,$$

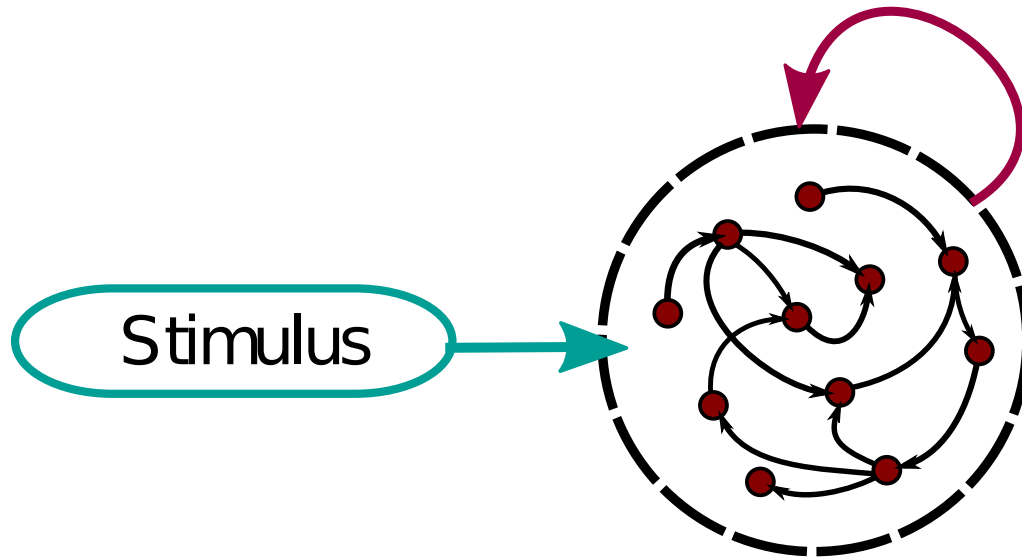
Match between model and experiment



Summary:

Synapses provide an additional layer of dynamics and computation, beyond that occurring in single neurons!

Network computation via simplified “firing rate” models



W , connection weight matrix

W_{ij} = weight from j to i

neuron (or neural population) i
fires with rate $r_i(t)$

neuron (or neural population) i
receives input

$$input_i = stim_i(t) + \sum_j W_{ij} r_j(t)$$

DYNAMICS: rates approach steady states $f(\text{input})$

$$\tau \frac{dr_i}{dt} = f(input_i) - r_i$$

